AN INVESTIGATION OF THE INVENTORY POLICIES OF A SMALL PRODUCTION FIRM

Richard Hanson Nace

NAVAL POSTGRADUATE SCHOOL

Monterey, California

DEPARTMENT OF THE NAVY ★ NAVAL POSTGRADUATE SCHOOL ★

THESIS

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by

Richard Hanson Nace

Thesis Advisor: F.R. Richards

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Richard Hanson Nace
Commander, Supply Corps, United States Navy
B.S., United States Naval Academy, 1957

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ABSTRACT

A study was conducted of the inventory operations of a small production plant. Based on parameters established or estimated during the study, a simple inventory model which yields stock levels for the raw material inventory and production quantities for the manufacturing operation was developed. Price comparisons were made showing the recommended policies to be more economical than those policies used before the study was conducted. A search routine proffered by Buffa and Taubert was examined and shown not to be acceptable as a method of determining production quantities for the company investigated.

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I. INTRODUCTION

A small manufacturing organization just as large corporations must make a myriad of inventory decisions in its daily operations. In a small organization these decisions may have a far greater impact on profit than do the identical decisions in a large firm. The decision and information flow and thus some degree of rationale is usually more readily evident in a smaller concern than in its larger cousin. Thus the inventory problem is often relatively more important, but at the same time easier to investigate in depth. Of course, the same principles are involved in both large and small operations; and findings from a study of a smaller system are readily adaptable to large systems -- even to an operation as large as the Department of Defense supply organization.

An inventory study was conducted at a small plant which produces self-propelled and non-self-propelled (i.e. trailers) cargo handling equipment. Since information of a proprietary nature was used in this study, it is felt that this material and information should be protected from disclosure to competing firms. Consequently, the production plant studied will, when necessary, be referred to simply as "The Company", and the products as models A, B, C, etc.

The Company maintains a spare part inventory which will be called "raw material" to provide direct support for the production effort; and to provide an immediately available source of parts for "parts sales" to customers who require

replacement of parts for previously purchased equipment. Annual part sales have been one million dollars while annual sales from their production effort has been approximately ten million dollars. At the time the study was conducted, the value of "raw materials" to support these sales was $1.45 million. This inventory had been reduced from $3.0 million the preceeding year because of continuing and subjective pressure from higher management in the parent firm. The reduction appears to be an over-reaction to high overhead costs. Coincident with this higher management pressure to reduce inventory levels, this writer was asked to investigate Company inventory policies in order to formalize a method of controlling the raw material and finished product inventories.

A second area of the inventory study concerned the equipments produced for stock. The Company produces twenty-three distinctly different items, many of which have a very low demand (in the order of two demands per year, or less) thus reducing any interest in maintaining a stock of those items. In addition, many of the items have a multitude of options that are available to the customer and must be built into the equipment during production. This factor could be likened to an automotive assembly line which produces many varieties of one type of automobile. This option problem as it relates to production is recognized as a fertile area for further study (i.e., expected options or expected sales, how they should effect the percent completion and the number

of the items to be produced strictly for stock). In view of the time limitations met in conducting this study, these models containing the various "built-in" options are not considered. However, four production items which are built either without options or with options that may be installed in the final steps of production (as the color of finish or size of tires in the production of automobiles) were considered. It should be noted, that most of the methods and principles used in discussing the four items in this paper would also apply to the other items produced by The Company.

II. DEVELOPING AN INVENTORY POLICY FOR PRODUCTION ITEMS

The present policy of The Company is to begin production of the item in the quantity of the order only after receiving a confirmed order for an item. Although that policy eliminates all holding costs of the production items, it forces a delay in satisfying all demands, it tends to maximize total set-up costs, and it prevents the planning of manpower resources in the production department.

Management personnel of The Company are not familiar with mathematical modeling techniques. There are sophisticated stochastic models of the production inventory process which have the benefits of providing a more nearly accurate representation of the actual operation. However, the problem of communicating their results, recommendations, and justifications take on an inordinate level of difficulty and thus lessen the confidence of the analyst in the eyes of management. If one assumes a continuous demand and constant demand rate, unit production costs that are not a function of the quantity produced (i.e. no learning is experienced during a production run), a constant production set-up cost independent of the size of the production quantity, and a holding cost rate solely dependent on the item cost, a simple model can be developed which utilizes an easy to follow logic flow (See chapter 2 of ref. 1). Since no quantitative analysis in this area had ever been attempted for The Company, even this simple model would provide a discernable step

towards reducing the costs of the operation. For this reason, the simple model was eventually settled upon to approximate the inventory operation. Further refinements, not only in demand and lead time distributions but in improving the accuracy of the parameters (holding cost rate, etc.) are recognized as worthy of further study. More will be said concerning parameter estimation later in this paper.

A. SET-UP AND UNIT COSTS

There is a cost connected with preparing the production line for full production. This is known as the "set-up" cost. A cost over and above the set-up cost, the unit cost must also be investigated. The set-up cost is primarily the labor cost (including base pay and benefits) of bringing the jigs and dies to the assembly line, clearing the production line for work, and positioning material. Since the cost of the jigs and dies are very small, the amortization costs were not considered.

Although The Company has an Industrial Engineering Division, it was not able to provide any direct measure of set-up costs. A partial list of detailed production function operator times and the related set-up times for only one of the four equipments under consideration was available. This information was used in a first attempt to establish set-up costs. This information, however, was found to be incomplete and judged unsuitable and too misleading to consider.

The Industrial Engineering Division has made the assumption that production operations are learned at a 95% rate. That is, when Q items are produced, the average cost of each of them, L_Q, is $.95^x$ times the cost of the item if only one were produced; where $x = \log_2 (Q)$. The assumption infers that each time production of a particular item starts, no learning from previous production runs of this item exists. The Industrial Engineering Department feels that since production workers are employed in manufacturing a wide variety of finished products and may not return to production of a previous item for a relatively long time, the carry-over of learning from one production run of the item to its next production run would be small.

In general, learning rates are applicable to operations (production of a particular item) and not to an individual or group (as a production division). Therefore, a constant learning rate, as in this case 95%, is not necessarily a valid assumption -- although it may approximate a median learning rate of the combined operations.

Recorded production costs for each order were available and were used in determining the total cost for each production run. Because of gross changes in production practices which occurred, no cost data over two years old was considered. The number of units made in the production run was also recorded. The following equations in two unknowns, A(set-up cost), and C(unit production cost, excluding set-up costs) were used to attempt to determine the two parameters:

$$T = (QL_QC) + A$$

where T = total recorded price for the production run

CL_Q = average unit cost for producing Q items

Q = number of items produced.

An example of a plot of total cost of a production run vs. quantity produced is shown in figure 1.

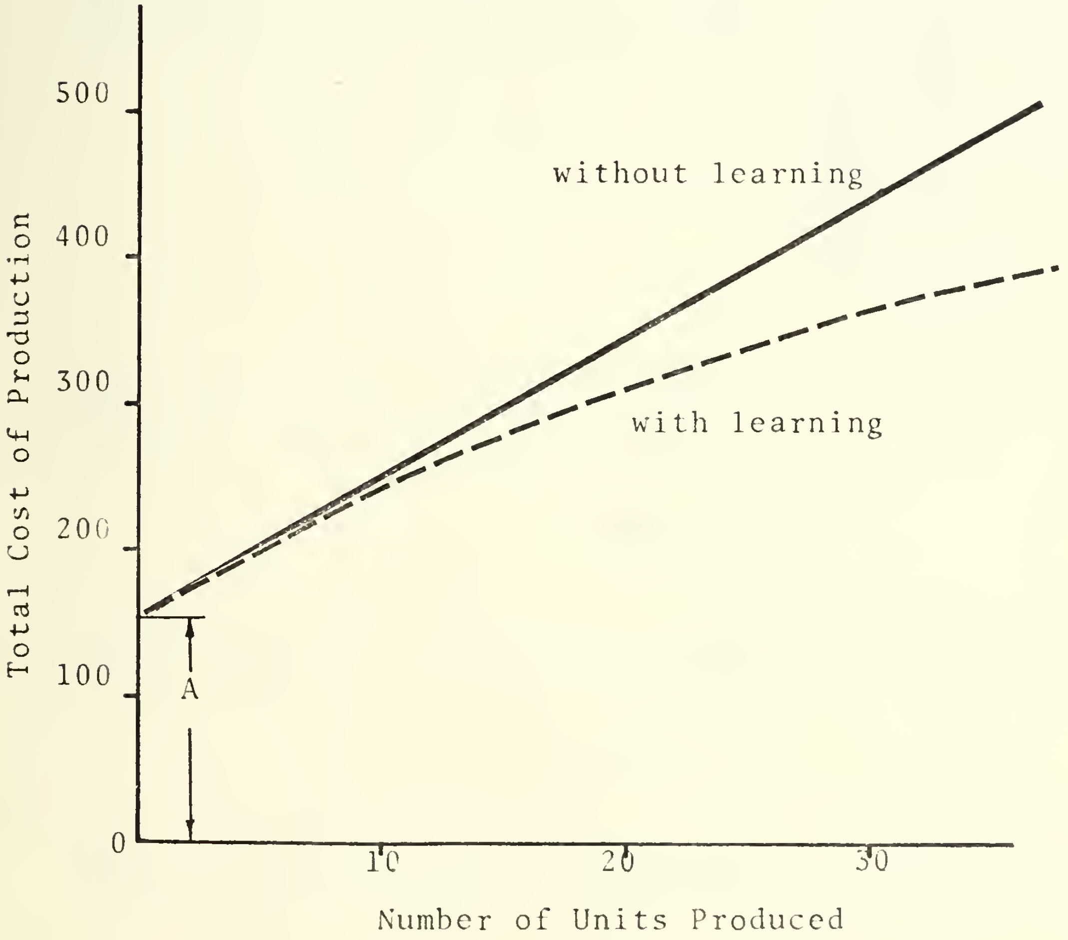

Fig. 1

The data points and computed parameters are displayed below for each model:

MODEL A

	computed parameters
data point 1: $Q = 25$; $T = \$179,400$	
data point 2: $Q = 50$; $T = \$267,450$	$C=\$2325$; $A=\$133,876$

MODEL B

	computed parameters
data point 1: $Q = 9$; $T = \$\ 9,882$	
data point 2: $Q = 30$; $T = \$\ 21,540$	$C_{12}=\$742$; $A_{12}=\$4232$
data point 3: $Q = 35$; $T = \$\ 22,925$	$C_{13}=\$667$; $A_{13}=\$4692$
data point 4: $Q = 12$; $T = \$\ 11,736$	$C_{14}=\$803$; $A_{14}=\$3736$
	$C_{23}=\$390$; $A_{23}=\$12,440$
	$C_{24}=\$735$; $A_{24}=\$4316$
	$C_{34}=\$415$; $A_{34}=\$7606$
	The means of the C_{ij}'s and the A_{ij}'s were computed as:
	$\overline{C}=\$627$; $\overline{A}=\$6171$

note: A_{ij} and C_{ij} are the computed A and C using data points i and j. Also $C_{ij} = C_{ji}$; and $A_{ij} = A_{ji}$.

MODEL C

	computed parameters
data point 1: $Q = 1$; $T = \$\ 13,672$	
data point 2: $Q = 6$; $T = \$\ 68,928$	$C = \$12,980$; $A=\$692$

MODEL D

	computed parameters
data point 1: $Q = 1$; $T = \$\ 25,599$	
data point 2: $Q = 10$; $T = \$179,627$	$C = \$20,750$; $A=\$4849$

The computed set-up costs in three instances were greatly different from those costs the management of The Company considered reasonable. The logic that The Company management used in arriving at their range of set-up costs seemed sound -- particularly when considering the simplicity of some of the items. The large discrepancy can be accounted to the manner in which the total costs of the production runs were accumulated. It is felt that inaccurate cost recording, particularly labor costs, and the restarting of production runs (i.e. a production run recorded as a single run of ten items may have been stopped and restarted several times thus incurring additional set-up costs which were not considered but were included in the total production costs) greatly distorted the accuracy and worth of the accumulated data.

No additional information was available to more accurately determine either set-up costs, unit costs, or learning rates.

A study with its attendant recommendations no matter how expertly carried out is worth nothing if its recommendations, if worthwhile, are not implemented. As a result of this, and being unable to prove or at least to adequately infer set-up and unit costs from the two attempts previously described, the costs used were those approximated by the Industrial Engineering Division and agreed upon as realistic estimates by the management of The Company. The set-up and unit costs used are as follows:

MODEL	SET-UP COST	UNIT COST (exclusive of set-up)
A	$ 1200	$ 5500
B	$ 57	$ 800
C	$ 2340	$ 12200
D	$ 3000	$ 20600

B. PRODUCTION RATE

The production rates were measured from recent historical data assuming no major material delays and average conditions and found to have the following values:

MODEL	PRODUCTION RATE
A	5 per week
B	20 per week
C	1 per week
D	1 per week for production runs less than five; 1.5 per week for production runs between five and ten; and 2 per week for production runs greater than ten.

note: The change in production run for model D is due to increased number of personnel normally assigned to the production effort as the production quantity is increased.

During the course of this study, the Production Manager often expressed his desires that the study not only provide a basis for making inventory control decisions but also provide methods that his employees may easily use to update the results as certain parameters change. A finite production rate does not lend itself to a simple updating procedure. Since this analysis was the first study made for The Company, ease of using the results was an important consideration. Furthermore, because of the fact that customers do not expect

immediate delivery of the products that they order, the actual production rate is not a critical parameter. In addition, the assumption of infinite production rates results in lower (i.e. conservative) economic production quantities - thus conforming with the general Company trend toward inventory reduction. As a result, it was decided to assume infinite production rates throughout the study.

C. DEMAND

Another parameter in the inventory model that must be considered is demand or expected demand. The Company's Sales Division prepares a listing of potential customers for the following twelve months. The sales predictions indicate the items and quantities, B_i, for which each customer is interested. In addition, the Sales Division estimates the probability, P_i, of making each sale. This is done for each customer, i. The expected sales of each item is the sum of the expected sales for each customer (i.e. $\sum_{\text{all } i} B_i P_i$).

The economic production quantity of all equipments herein considered is a direct function of expected sales, and accurate sales forecasting takes on a high degree of importance. Since expected sales was the only forecast data available and represented The Company's best effort in this area, they were assumed to be within acceptable bounds of accuracy.

The nature of the items under consideration is such that the customers anticipate a delay before their orders will be

received. Because of this, it can be assumed that backorder costs are zero as long as the item is delivered within an expected production lead time plus delivery time. If The Company were to stock production products based on anticipated demands, they would be able to provide more rapid response to customer's orders. How this additional service would result in increased sales ; how the stocked items might affect sales promotions; and how impulse buying might increase sales can only be considered an additional but unaccountable benefit from this study. In fact, this area itself is a field worthy of a separate study.

D. PERIODIC REVIEW AND THE SEARCH DECISION RULE

In the initial attempt to provide management with a method to decide on a policy to govern production, a dynamic periodic review model was developed. Since The Company has forecasts of the next four quarters of demands and no backorder costs are suffered provided all demands are filled within a reasonable amount of time, the following assumptions were made:

(1) The demands in the next four quarters are known to be F_{i1}, F_{i2}, F_{i3}, F_{i4} where F_{ij} is the forecast of the total sales of model i in quarter j.

(2) Each production run commences at the beginning of the quarter and will be completed before the beginning of the next quarter regardless of the amount produced.

(3) The startup cost for each production run for Model i is A_i, independent of the quantity to be produced.

(4) The holding cost in quarter j for model i is $I_i C_i S_{ij}$ where I_i is the annual holding cost rate for model i, C_i is the unit cost of model i and S_{ij} is the average value of the on-hand inventory of model i in quarter j.

(5) All demands must be satisfied within the quarter.

(6) The total cost of producing Q_{ij} units of item i in quarter j is $A_i \delta_{ij} + C_i Q_{ij}$ where $\delta_{ij} = \begin{cases} 0 \text{ if } Q_{ij} = 0 \\ 1 \text{ if } Q_{ij} > 0 \end{cases}$

The objective is to determine, for each item i and quarter j, that quantity $Q_{ij} \geq 0$ to produce in order to minimize total production and holding costs subject to the constraint that all demands be satisfied within a quarter. Let N_{ij} be on-hand inventory for item i at the beginning of quarter j. Then the problem can be written:

$$(1) \quad \min_{Q_{ij}} \; K = \sum_{i=A}^{D} \sum_{j=1}^{4} (A_i \delta_{ij} + C_i Q_{ij} + I_i C_i S_{ij})$$

$$\text{subject to } N_{ij} + Q_{ij} \geq F_{ij} \quad \text{for } i=A,B,C,D \text{ and } j=1,2,3,4$$

note: The inclusion of the term $A_i \delta_{ij}$ causes the program to fail to be linear.

To solve this production planning problem an approach proferred by E. Buffa and W. Taubert (ref. 2) which is presently enjoying widespread attention was considered. Their approach, called the Search Decision Rule, SDR, attempts to solve aggregate planning problems through the use of optimum-seeking computer search methods. The heart of the Search Decision Rule is the computer search routine provided by Buffa and Taubert. To use this technique, a computer program was written which provides initial conditions, bounds on the

decision variables, and a function which determines the cost for any set of values of the decision variables. In order to handle the stockout constraint, the penalty function approach was used. In particular, each time $F_{ij} > N_{ij} + Q_{ij}$, a penalty function of 10^9 dollars was added to the value of equation (1). This quantity is of such magnitude that in minimizing costs, the constraint would never be violated. Beginning with the input values for the decision variables, SDR, the search subroutine, calculates the value of the objective function and then attempts to decrease this value by modifying the decision variables. This process continues until the search subroutine fails to recognize an improvement in the value of the objective function. The last set of values of the decision variables represents the recommended production quantities. As with any search technique, SDR does not guarantee optimality but it is expected that the values obtained provide at least a local minimum.

The Search Decision Rule was tested several times using actual Company data. In many of the cases a simple visual inspection revealed that the results could be improved. This was felt to be a result of the initial conditions. Therefore several sets of initial conditions were examined and the results indicated that the search technique is overly sensitive to the initial values. In fact, the technique is so severely sensitive to the initial values that it was judged to be inadequate for solving the production planning problem.

After rejecting the Buffa-Taubert technique as a tool for obtaining optimal values for the periodic review model; dynamic programming was considered. Note that the objective function is separable in the sense that it is the sum of the costs incurred for each of the four models. Define $Z_{in}(\xi_i, Q_{in})$ to be the minimum cost over the first n quarters of operation for model i when the on-hand inventory at the end of quarter n is ξ_i and the quantity produced is Q_{in}. Then let

$$Z_{in}(\xi_i) = \min_{Q_{in} \geq 0} Z_{in}(\xi_i, Q_{in}).$$

The following recursive relation is derived:

$$(2)\quad Z_{in}(\xi_i) = \min_{Q_{in} \geq 0} \{A_i \delta_{in} + C_i Q_{in} + I_i C_i [\xi_i + \frac{F_{in} - Q_{in}}{2}] + Z_{i,n-1}(\xi_i - Q_{in} + F_{in})\}$$

It was observed that equation (2) is the same as that developed by Hadley and Whitin in Section 7-4 of ref. 1. They develop a simple algorithm for determining the values of the decision variables which are optimal. The algorithm shows that the optimal production quantity for any quarter is always an integral number of future quarters demand. For example, if n = 3, $Q_{i3}^* = 0$ or F_{i3} or $F_{i3} + F_{i4}$. The optimal value is selected by considering the holding costs and set-up costs over this small range of alternatives.

These results were applied with the parameters previously discussed, and the actual Company forecasts for a four quarter period. The optimal production quantities for each

of the models over the four quarter planning horizon were calculated for a range of reasonable values of the holding cost rate. In each case, the production quantities were found to be those values which satisfy only the current quarterly demand.

This approach shows that forecasts of demand for quarters other than the current quarter are not important in determining the current production quantity. Since it only allows production at the beginning of each quarter, it seems possible the costs could be reduced even more if production were allowed at any time within the quarter. In order to examine this possibility a continuous review model is considered.

E. HOLDING COSTS AND ECONOMIC PRODUCTION QUANTITY

Holding cost consists of those costs which are related to keeping material in stock. It includes such items as wages of personnel related to receiving, storing, and issuing the material, the cost of money tied up in the inventory, rental of buildings, etc. used for holding inventory, taxes directly charged on the inventory value, and deterioration and obsolescence of the material kept in inventory. Those factors of holding costs caused by product deterioration during periods of unprotected outside storage, and obsolescence of the item as the industry needs vary are considered to be substantial and yet are very nebulous to price. It is usually the experience in a study of this type that

company's management feels it can adequately estimate holding cost. Therefore a matrix, one for each item under study, was developed which enables management to adapt to changing holding costs and presents a set of alternatives for management to choose among. The columns are a range of reasonable holding cost rates (in percentages). Management may select the most "reasonable" holding cost rate. This decision determines a row and therefore also a production quantity, Q, which is read from the left-hand column. In computing the production quantity, it was assumed that the demands were deterministic and known with annual rate λ, the set-up costs, A, were constant, no back orders were allowed, production rate was infinite, unit cost, C, is independent of quantity produced, and the holding cost was a function only of inventory value and accrued at an annual rate, I. The total annual cost, K, is as follows:

$$K = \frac{A\lambda}{Q} + \frac{ICQ}{2} + CQ$$

Considering only integer production quantities, Q, the minimum cost is achieved when

$$Q(Q-1) = \frac{2A\lambda}{IC}$$

It was the method that was used to determine the production quantity for models A,B,C, and D. The unit and set-up costs for each unit is given in section A of this paper. The results of these computations appear as tables I,II,III and IV immediately following this section.

PRODUCTION TABLE FOR MODEL A

ANNUAL DEMAND

Q	2	4	5	10	15	20	25	30	35	40	50	60	70
1	- - - - - - - - - - - - INFINITELY LARGE - - - - - - - - - - - - - - - -												
2	43.6	87.2	109	218	327	436	546	655	764	872	1091	1309	1527
3	14.5	29.1	36.4	72.7	109	145	182	218	254	291	364	436	509
4	7.3	14.6	18.2	36.4	54.6	72.6	91.0	109	127	146	182	218	255
5	4.4	8.7	10.9	21.8	32.7	43.6	54.5	65.4	76.3	87.2	109	131	153
6	2.9	5.8	7.3	14.6	21.8	29.1	36.4	43.6	50.9	58.2	72.3	87.3	101.8
7	2.1	4.2	5.2	10.4	15.6	20.8	26.0	31.2	36.4	41.6	51.9	62.3	72.7
8	1.6	3.1	3.9	7.8	11.7	15.6	19.5	23.4	27.3	31.2	39.0	46.8	54.5
9	1.2	2.4	3.0	6.1	9.1	12.1	15.2	18.2	21.2	24.2	30.3	36.4	42.4
10	1.0	1.9	2.4	4.8	7.3	9.7	12.1	14.5	17.0	19.4	24.2	29.1	33.9
11	.8	1.6	2.0	4.0	5.6	7.9	9.9	11.9	13.9	15.9	19.8	23.8	27.8
12	.7	1.3	1.7	3.3	4.9	6.6	8.3	9.9	11.6	13.2	16.5	19.8	23.1

TABLE I

PRODUCTION TABLE FOR MODEL B

ANNUAL DEMAND

Q	1	5	10	15	20	25	30	35	40	45	50	55	60	65	70
1	-INFINITELY LARGE-														
2	5.8	29.2	58.5	87.7	117	146	175	205	234	263	293	322	351	380	410
3	1.9	9.7	19.5	29.2	39.0	48.7	58.5	68.2	78.0	87.7	97.5	107	117	127	137
4	1.0	4.9	9.7	14.6	19.5	24.4	29.2	34.1	39.0	43.9	48.7	53.6	58.5	63.4	68.2
5	.6	2.9	5.8	8.8	11.7	14.6	17.5	20.5	23.4	26.3	29.2	32.2	35.1	38.0	40.9
6	.4	1.9	3.9	5.8	7.8	9.7	11.7	13.6	15.6	17.6	19.5	21.4	22.4	25.3	27.3

TABLE II

PRODUCTION TABLE FOR MODEL C

ANNUAL DEMAND

Q	1	2	3	4	5	6	7	8	9	10	15	20	25	30	35
1	- - - - - - - - - - - - - - - - -INFINITELY LARGE- - - - - - - - - - - - - - - - - -														
2	19.2	38.4	57.5	76.7	95.9	115.1	134	153	172	192	287	383	479	575	671
3	6.4	12.7	19.2	25.6	32.0	38.4	44.7	51.2	57.5	63.9	95.9	127.9	160	192	224
4	3.2	6.4	9.6	12.8	16.0	19.2	22.3	25.6	28.8	32.0	47.9	63.9	80.0	95.9	112
5	1.9	3.8	5.8	7.7	9.6	11.5	13.4	15.3	17.3	19.2	28.8	38.4	47.9	57.5	67.1
6	1.3	2.6	3.8	5.1	6.4	7.7	8.9	10.2	11.5	12.8	19.2	25.6	32.0	38.4	44.7
7	.9	1.8	2.7	3.6	4.6	5.5	6.4	7.3	8.2	9.1	13.7	18.3	22.8	27.4	32.0
8	.7	1.4	2.1	2.7	3.4	4.1	4.8	5.5	6.2	6.8	10.3	13.7	17.1	20.6	24.0

TABLE III

PRODUCTION TABLE FOR MODEL D

ANNUAL DEMAND

Q	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	- - - - - - - - - - - - - - - - -INFINITELY LARGE- - - - - - - - - - - - - - - - - - -														
2	29.1	43.6	58.2	72.7	87.3	102	116	131	146	160	175	189	204	218	233
3	9.7	14.5	19.4	24.2	29.1	33.9	38.8	43.6	48.5	53.3	58.2	63.1	67.9	72.7	77.6
4	4.8	7.3	9.7	12.1	14.6	17.0	19.4	21.8	24.2	26.7	29.1	31.5	33.9	36.4	38.8
5	2.9	4.4	5.8	7.2	8.7	10.2	11.6	13.1	14.6	16.0	17.5	18.9	20.4	21.8	23.3
6	1.9	2.9	3.9	4.8	5.8	6.8	7.7	8.7	9.7	10.7	11.6	12.6	13.6	14.5	15.5

TABLE IV

Attention must be called to the fact that the unit costs are constant and not a function of the number produced in each production run. In effect this ignores the fact that learning takes place during multiple unit production and gives a smaller or more conservative economic production quantity. It would be a simple matter, however, to account for learning if it were felt that one knew the learning rate. For example, under the constant learning rate discussed previously, one would modify the entries in the matrix by a multiplier as follows:

$$\frac{I_U}{I_L} = \frac{\dfrac{2A\lambda}{C[Q(Q-1)]}}{\dfrac{2A\lambda}{L_Q[Q(Q-1)]}} = \frac{L_Q}{C}$$

or

$$I_L = I_U \frac{C}{L_Q}$$

where: L_Q is the average unit cost when producing Q items

I_L is the holding cost rate considering learning

I_U is the holding cost rate ignoring learning which is shown in the following matrices.

The simplification of ignoring learning rates was made for the reasons: 1) Company personnel will be better able to update the matrices as changes or refinements in model parameters are made; 2) the "no-learning" gave more conservative production decisions which conform with previously discussed Company policy of reducing inventory investment; and 3) it was felt that the true learning function was not known.

F. STOCKING POLICY

The present stocking policy for all produced equipments of The Company allows stockouts. As has been discussed earlier, customers anticipate a normal production and shipping time delay after submitting their orders. Therefore, stockout costs during the period have been ignored in the determination of the production quantities. Although stockout costs were not considered, it must be acknowledged that they do exist and their cost is in fact greater than zero. One can infer that inventory stock levels should be revised (thus reducing stockouts) to minimize the total costs related to production inventory operations.

The recommended production policy for each model is to produce at the time the order is received either the quantity, Q, determined for that model, or produce the quantity ordered less the amount on hand, whichever is greater.

G. COST COMPARISON

It cannot be shown that the recommended stocking policy is optimal. However, to establish the worthiness of the production quantity recommendations provided herein, a twelve month comparison was made between the costs incurred if policies of The Company were followed and the costs incurred if the recommendations of this paper were followed.

Annual demand estimates which were available during the sample period were used to determine production quantities.

The annual sales (or demand estimates) were updated quarterly thus allowing production guidelines, Q, to also be updated. Utilizing the production quantity, Q, for the current quarter, production of items was initiated upon the occurrence of the first demand. The annual holding cost rate, I, was assumed to be 0.33. This rate is considered to be an upper bound on the parameter and was selected to demonstrate the effectiveness of this model even under such an adverse environment. The items were reduced from inventory as actual sales, as indicated by shipping data, occurred. When the inventory became zero and a demand was unfilled (i.e. a backorder existed) the recommended production quantity (or a minimum integer multiple of this quantity) was produced to at least meet all current demands. As an example, suppose an inventory of five existed, and based on the recent estimates of annual demand the recommended production quantity was three. If a demand of nine were received, the five in inventory would be issued and six would be produced to meet the outstanding demand of four. The inventory remaining after this transaction would be two (5-9+6). However, only one set-up cost would be incurred since only one production run was experienced.

Accumulating holding costs over the year and adding set-up costs for each production run, the total inventory costs for each model was determined. This was compared to the cost incurred as a result of the current Company policy of making the quantity demanded upon receipt of the demand (thus

eliminating any holding costs). It must be noted here that the single exception to the current general Company policy is for the production of model A when The Company always produces in batches of twenty-five. This exception is considered in the cost computations below. The results of this comparison immediately follow:

		Recommended Policy			Company Policy		
	Actual Demand	Holding Cost	Set-up Cost	Total Cost	Holding Cost	Set-up Cost	Total Cost
Model A	77	$6680	$12000	$18680	$28404	$ 4800	$33204
Model B	16	$ 280	$ 170	$ 450	0	$ 227	$ 227
Model C	14	$6600	$ 9360	$15960	0	$16380	$16380
Model D	7	$7525	$ 9000	$16525	0	$18000	$18000
			Total Cost	$51615		Total Cost	$67811

With the exception of Model B, the decisions recommended resulted in significant savings in costs (a total of 24% annually) over those actually incurred using The Company's policy. If backorder costs were considered in this comparison, the savings would have been even greater. Likewise if the value of I in the comparison were reduced to a level which is probably more realistic, the resultant savings would have been even more significant.

The apparent failure of the recommended production policy to reduce inventory costs for Model B was due to a very erratic demand over the period, and the fact the sales estimates were very poor. Since the production quantity depends heavily on the forecasted values, these results suffered in the comparison for Model B. Using these sales estimates caused an

unnecessary inventory buildup and the low and erratic realization of actual sales resulted in a surplus of stock. This is a very good example of the importance of sales estimates as used in this model and as earlier discussed in this paper. This indicates that perhaps one should calculate λ as four times the current quarter's demand instead of using the sum of the next four quarter's demand. Had this been done, the example just examined would have displayed even more savings. It is felt that these cost savings fully support the recommendations made in this paper.

H. DISCUSSION OF MODEL WEAKNESSES

Several weaknesses in parameter estimation have already been mentioned in developing the model for production equipment inventory decisions. It seems appropriate to proffer some suggestions for making improvements in these parameters.

The parameters having least credibility are those concerning set-up costs, unit costs, and learning rates. Several avenues of approach toward better estimating them are available.

The total cost, T, of the production run which produces a quantity, Q, is given by:

$$T = QL_QC + A$$

where $L_Q = (LR)^{\log_2(Q)}$ and LR is the unknown learning rate. The equation is in three unknwons, C, A, and LR. If it were possible to achieve exact data as to the total cost, T, of each run, three production runs of three separate production

quantities could be made and the three equations solved for the unknowns. Since exact data accumulation is not expected, particularly with the stochastic nature of the cost itself, other alternatives are considered more practical.

The Industrial Engineering Division could compile a study covering detailed production functions for every facet of the production process for the four equipments. From this information, each action in the process could be assigned "set-up" or "production" phases of the operation. By using already established labor hourly costs multiplied by set-up time hours , the applicable set-up costs could then be estimated with a higher degree of accuracy. Using existing total cost data for single and multiple item production runs, unit costs and learning rates could be more closely estimated.

The third method of determining these parameters assumes a known learning rate. Several production runs of various quantities are made and their total costs, T, recorded. For each pair of production run costs, determine the value of A and C from:

$$A = T - QL_Q C$$

$$C = \frac{T - A}{QL_Q}$$

One thus obtains n(n-1) values for A and C (where n is the number of production runs). Determine the variance of both A and C. Repeat this procedure for several feasible learning rates finally choosing that learning rate and mean values of A and C for which the variances of A and C are minimum.

III. DEVELOPING A POLICY FOR RAW MATERIAL INVENTORY MANAGEMENT

A. MATERIAL CATEGORIES

To support both the production operation and sales of spare parts, a raw material inventory of approximately 11,000 line items is maintained. In order to attempt to better concentrate management attention, The Company has established three material categories and related stocking policies.

1. Items whose Unit Costs are Less than One Dollar

These items are stocked without stock record cards, but are maintained in stock on a "two-bin" or "low level" system. That is, when the stock reaches a predetermined low level (generally signified by the opening of a specially marked box or set of boxes containing the remaining items in stock) an order to raise the item's inventory position to its predetermined high level, is initiated. The high level is reviewed periodically by the stock control supervisor. This policy, commonly known as an (S,s) policy, has been widely investigated and found to work well in many situations.

2. Items whose Unit Cost Range from One to Five Dollars

Under current policy stock record cards are maintained for these items. Orders are initiated when the stock level falls below a predetermined low limit and the quantity ordered is that amount which raises the inventory position to a predetermined high level. The high and low levels were

determined subjectively after considering recently recorded demand so as to minimize stock-outs while keeping inventory levels to within "reasonable" limits.

3. Items whose Unit Cost is Greater Than Five Dollars

Although stock record cards are maintained for these items, the items are generally not kept in stock. When an item is required, it is ordered only in the amount required to meet the current need. In a relatively few cases, as when demands are very high, the items are stocked as mid-range priced items (i.e. inventory levels are maintained between high and low limits).

B. BACKGROUND OF MODEL DEVELOPMENT

A deterministic model like that used to establish production quantities was also used to establish order quantities for each of the three categories of raw material. It is recognized that the demand is not deterministic, but random; however, attempts to determine the distribution of these demands was unsuccessful. The data showed that items were usually demanded in batches. This led to a test that the demands were distributed as a compound Poisson. A Chi-squared goodness-of-fit test indicated strongly that the customer inter-arrival time was not exponentially distributed. As a simplifying assumption order quantities were calculated based on deterministic demand, but reorder levels were dependent upon the stockastic nature of the actual demand history.

C. MODEL PARAMETERS

It was felt that an accurate figure for the holding cost rate for production items could not be obtained for lack of information primarily about obsolescence and deterioration. However, these two factors were not as important when applied to the raw material inventory. A figure of 14.5% per year per dollar of inventory cost was determined. This rate consisted of a 6% charge imposed by the parent company; 1/2% for warehouse rental; 2.7% for salary and fringe benefits of employees connected with maintenance and operation of the inventory function, 3% for taxes on the inventory itself, and an additional 2.3% The Company feels it should consider as a "cost of money" charge over and above the 6% charged by the parent company. Insurance is paid by the parent company and is not considered a function of raw material on-hand inventory value. Intangibles such as pilferage, obsolescence and deterioration were not directly considered in determining holding cost rate.

Purchase cost, A, was found to be $16.00 per order. This was computed from the costs of material and services (including telephone service) used in the purchasing operation, and from the wages and benefits paid to personnel of the purchasing division and to those in the accounting division concerned with processing and paying for those procurements. The total of these charges were approximately $8000 for an average month and an average months orders numbered

512. Thus:

$$A = \frac{\$8000}{515} \approx \$16.$$

It appears that both the holding cost rate and the procurement costs are lower than what is actually experienced at other similar industries. Note that the expression for the optimum order quantity derived from the simple model $[2\lambda A/IC]^{1/2}$, depends on parameters A and I only through ratio A/I. Although the values obtained as estimates of these parameters both seem to be smaller than those anticipated, it is felt that the errors of the estimates are not critical since the magnitude of the ratio seems to be relatively good.

The parameters of unit cost, C, and annual demand, λ, must also be considered. The information on the stock record cards (using current Company posting procedures) will always reflect the last unit cost. Any large changes in this value will be immediately evident and require a recalculation of the order values (see next section). Changes in annual demand may also occur. Like the changes in unit cost, this information is also on the stock record card. It is not displayed directly but must be computed periodically. By considering the changes in these two parameters, the model will readjust itself to current conditions.

D. ORDER QUANTITIES

A matrix shown on the following page was constructed based on the A/I ratio discussed in section C. The matrix

UNIT COST ($)

λ DEMAND	.01	.10	.25	.50	1	2	4	5	8	10	15	20	30	40	50	75	100
3	257	81	51	36	26	18	13	11	9	8	7	6	5	4	4	3	3
4	297	94	59	42	30	21	15	13	10	9	7	6	6	4	4	3	3
6	364	114	73	52	36	25	18	15	13	12	9	7	6	6	4	4	3
9	446	141	89	62	45	31	22	19	16	13	12	10	7	7	6	4	4
12	514	162	103	73	52	36	25	24	18	16	13	10	9	7	7	6	4
15	575	181	114	82	58	40	28	25	21	18	15	13	10	9	7	6	6
18	630	199	126	89	62	45	31	28	22	19	16	13	12	10	9	7	6
21	680	215	137	97	68	48	34	30	24	21	18	15	12	10	9	7	7
24	728	230	146	103	73	52	37	33	25	22	19	16	13	12	10	9	7
30	814	257	163	114	82	58	40	36	27	25	21	18	15	13	12	9	7
36	891	282	178	126	89	62	45	40	31	28	22	19	16	13	13	10	9
48	1029	325	206	146	103	73	52	46	36	33	27	22	19	16	15	12	10
60	1151	364	230	163	114	82	58	52	40	37	30	25	21	18	16	13	12
72	1261	398	253	178	126	89	62	56	45	40	33	28	22	19	18	15	12
84	1362	431	272	193	137	97	68	61	48	43	36	30	24	21	19	16	13
96	1456	461	291	206	146	103	73	65	52	46	37	33	27	22	21	16	15
108	1543	489	309	218	154	108	77	68	55	49	40	34	28	24	22	18	15
120	1627	514	325	230	163	114	82	73	58	52	42	36	30	25	24	19	16
240	2300	728	461	325	230	163	114	103	82	73	59	52	42	36	33	27	22
600	3640	1151	728	514	364	257	181	163	129	114	94	82	67	58	52	42	36

ORDER QUANTITIES

TABLE V

shows the economic order quantities (EOQ) for the range of unit costs and annual demands experienced by The Company.

E. REORDER LEVELS

Although a method for determining the order quantities, assuming deterministic demands and no stockout costs has been recommended, one must not ignore either the random nature of the demand process or the stockout costs. These are both considered in the determination of the reorder levels. To account for these factors, the distribution of demand during leadtime for each item must be considered. Define the safety level for an item to be the expected stock on hand when a shipment arrives. Then to reduce the probability of a stockout, it would be necessary to increase the safety level and in this manner provide protection against unanticipated higher demands. As the critical interval of time is the period between the placing of an order and the receipt of that order, it is the distribution of leadtime demand that must be considered. Many studies have shown that a Normal distribution for leadtime demand (especially in the case of high demand items) is a good approximation of the distribution usually experienced in these cases. However, for illustration purposes, consider the empirical distribution of leadtime demand for a representative item shown in figure 2. From this empirical distribution, the expected leadtime demand is found to be five units. Therefore, if an order is placed when the stock on hand reaches five, the safety

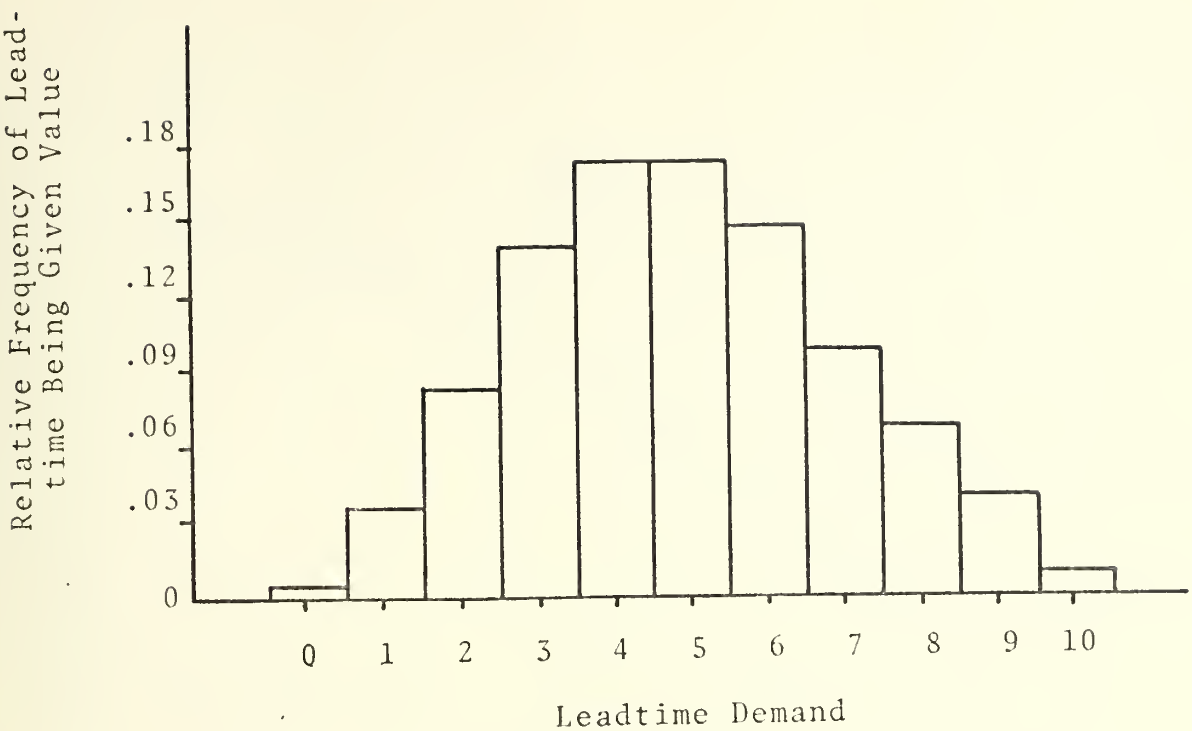

Fig.2

level is zero and the probability of a stockout occurring, P(OUT), would be .56, that is, the leadtime demand would exceed five with probability .56. Similarly, for a reorder level of ten (a safety level of five) the probability of being out of stock would fall to approximately P(OUT) = .03. For the above example, the probability of incurring a stockout for various safety levels is shown in fig. 3.

As a unit change in safety level is made, a change in the inventory holding costs equal to IC is incurred. These changes are shown in fig. 4 for items with several unit costs. Thus it is recommended that management balance the desired service level (i.e. the probability of incurring a stockout) with the increased holding cost in determining

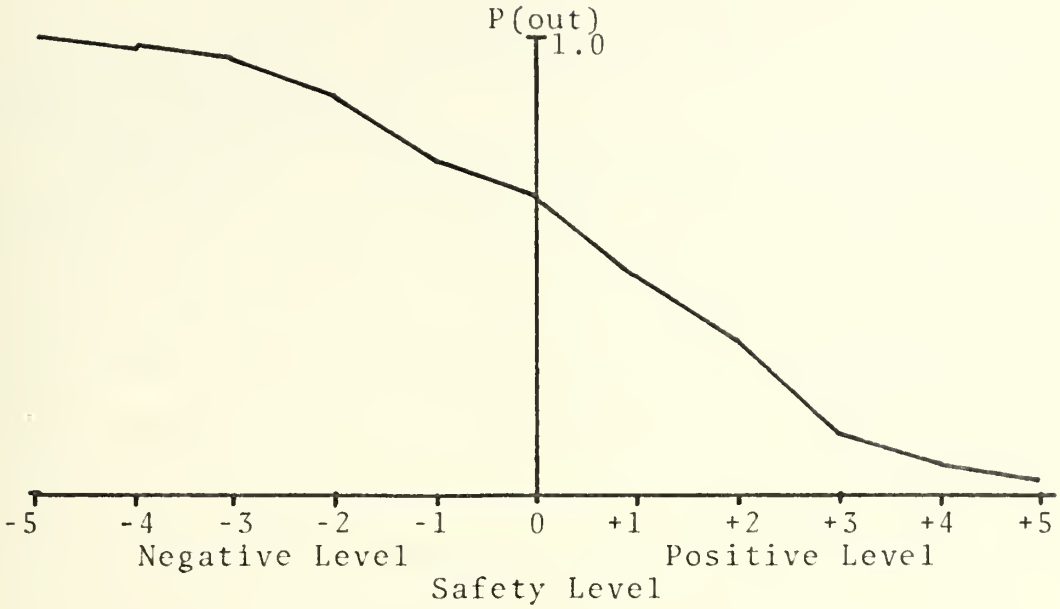
P(out)
1.0
-5
-4
-3
-2
-1
0
+1
+2
+3
+4
+5
Negative Level
Positive Level
Safety Level

Fig. 3

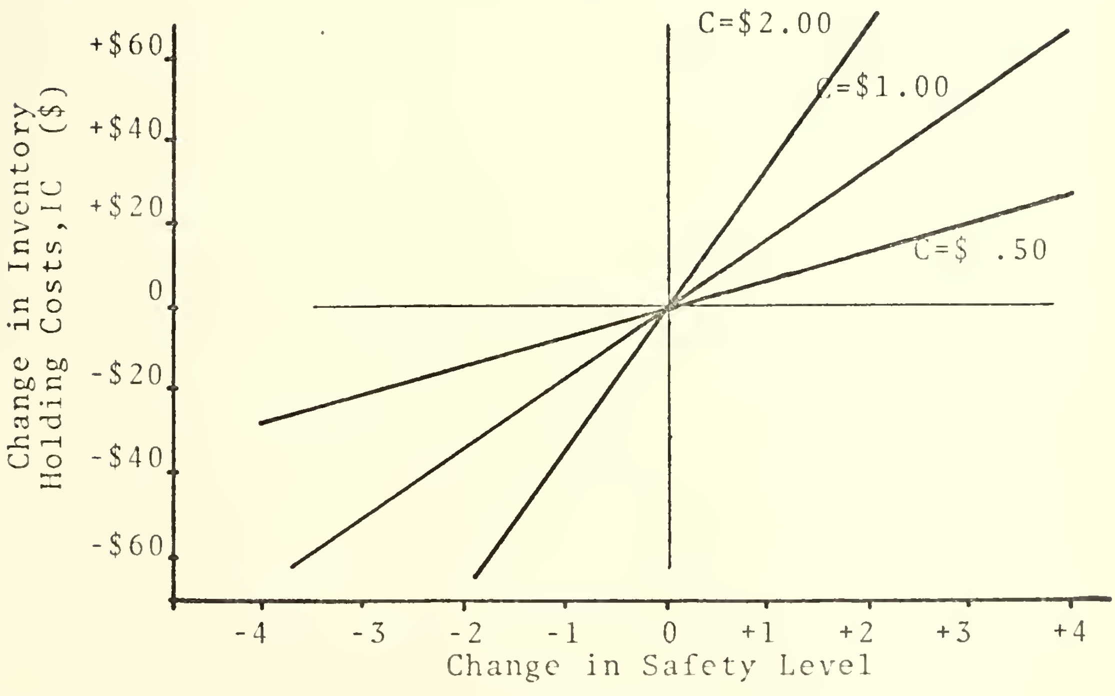
Change in Inventory
Holding Costs, IC ($)
+$60
+$40
+$20
0
-$20
-$40
-$60
C=$2.00
C=$1.00
C=$ .50
-4
-3
-2
-1
0
+1
+2
+3
+4
Change in Safety Level

Fig. 4

safety stock. It should be noted that since the unit increase in holding cost per unit increase in safety level equals IC, the more expensive the item is, the smaller the safety level should be.

Another way to view the importance of balancing the extra holding cost with the increase in service is to consider the extra holding cost as a stockout cost, for it is an extra cost that is paid to reduce stockouts. Therefore an imputed stockout cost can be determined for any given safety level by comparing the total holding cost for a given safety level, S, with the total holding cost when a safety level of zero is maintained. The differences between the two costs is given by ICS. These imputed stockout costs are shown in fig. 4 for the example considered.

COMPUTER PROGRAM

```
C
C THIS IS THE BUFFA-TAUBERT SDRMIN SUBROUTINE COUPLED WITH THE
C  MAIN PROGRAM AND THE OBJFCT SUBROUTINE USED AND DISCUSSED IN
C  THIS THESIS.
C
C
C
C
         MAIN PROGRAM
      COMMON X(24),C(24),H(24),S(24),SU(24),XNV(24),D(24),BNIS(24),RIALV
     1(24),XMAX(24),XMIN(24),N,MAXTRY,TRIALV
C..........THE NUMBER OF DECISION VARIABLES EQUALS "N"
      N=24
C..........THE EXPECTED DEMANDS ARE ENTERED D(I) WHERE EACH
C             REPRESENTS A DEMAND FOR ONE OF THE FOUR MODELS
C             ACCORDING TO THE FOLLOWING MATRIX:
C
C        PER. 1     PER. 2     PER. 3     PER. 4     PER. 5     PER. 6
C    A      1          5          9         13         17         21
C    B      2          6         10         14         18         22
C    C      3          7         11         15         19         23
C    D      4          8         12         16         20         24
C
      D(1)=3.95
      D(2)=0.0
      D(9)=3.9
      D(13)=6.75
      D(5)=12.3
      D(6)=25.
      D(10)=25.
      D(14)=113.5
      D(3)=0.0
      D(7)=1.75
      D(11)=0.0
      D(15)=0.0
      D(4)=0.0
      D(8)=3.5
      D(12)=0.
      D(16)=0.0
      DO 567 I=17,21,4
      D(I)=D(13)*1.1
      D(I+1)=D(14)*1.1
      D(I+2)=D(15)*1.1
  567 D(I+3)=D(16)*1.1
      MAXTRY=2000
      DO 3019 I=1,N
C..........SETTING MAX. AND MIN. VALUES FOR THE DECISION VARIABLES
      XMAX(I)=800.
 3019 XMIN(I)=0.
```

```
C..........SETTING INITIAL VALUES OF THE DECISION VARIABLES
      X(1)=4.07
      X(5)=12.3
      X(9)=3.9
      X(13)=6.75
      X(17)=7.4
      X(21)=7.4
      X(2)=0.
      X(6)=25.
      X(10)=25.0
      X(14)=115.
      X(18)=126.5
      X(22)=126.5
      X(3)=0.0
      X(7)=0.0
      X(11)=0.0
      X(15)=0.0
      X(19)=0.0
      X(23)=0.0
      X(4)=0.0
      X(8)=3.5
      X(12)=0.0
      X(16)=0.0
      X(20)=0.0
      X(24)=0.0
C..........XNV(I) IS THE INVENTORY AT THE START OF PERIOD (I)
C            BEFORE PRODUCTION DURING THAT PERIOD HAS STARTED
      XNV(1)=10.
      XNV(2)=26.
      XNV(3)=0.
      XNV(4)=7.
C..........H(I) IS THE HOLDING COST RATE
      H(1)=.33
      H(2)=.33
      H(3)=.33
      H(4)=.33
C..........C(I) IS THE PRODUCTION COST OF EACH OF THE MODELS
      C(1)=5500.
      C(2)=800.
      C(3)=12200.
      C(4)=20600.
C..........S(I) IS THE SET-UP COST OF EACH OF THE FOUR MODELS
      S(1)=1200.
      S(2)=56.8
      S(3)=2340.
      S(4)=3000.
      DO 3001 I=1,4
      DO 3002 K=4,20,4
```

```
      H(I+K)=H(I)                                                       MIN00950
      C(I+K)=C(I)                                                       MIN00960
 3002 S(I+K)=S(I)                                                       MIN00970
 3001 CONTINUE                                                          MIN00980
      CALL SDRMIN                                                       MIN00990
      STOP                                                              MIN01000
      END                                                               MIN01010


      SUBROUTINE OBJFCT                                                 MIN01020
      COMMON X(24),C(24),H(24),S(24),SU(24),XNV(24),D(24),BNIS(24),RIALVMIN01030
     1(24),XMAX(24),XMIN(24),N,MAXTRY,TRIALV                            MIN01040
      DO 3021 I=5,24                                                    MIN01050
C...........DETERMINING THE INVENTORY AT THE START OF EACH PERIOD       MIN01060
 3021 XNV(I)=XNV(I-4)+X(I-4)-D(I-4)                                     MIN01070
      DO 3030 II=1,N                                                    MIN01080
      ASSET=XNV(II)+X(II)                                               MIN01090
      BNIS(II)=0.                                                       MIN01100
      IF(ASSET.LT.D(II)) BNIS(II)=1000000000.                           MIN01110
  211 IF(X(II).LT.0.0001) GO TO 220                                     MIN01120
      IF(X(II).GE.0.0001) GO TO 221                                     MIN01130
C...........IF NO ITEMS ARE PRODUCED, NO SET-UP COSTS ARE INCURRED      MIN01140
  220 SU(II)=0.                                                         MIN01150
      GO TO 3030                                                        MIN01160
C...........IF ANY ITEMS ARE PRODUCED, THE SET-UP COST IS INCURRED      MIN01170
  221 SU(II)=S(II)                                                      MIN01180
 3030 CONTINUE                                                          MIN01190
      TRIALV=0.                                                         MIN01200
C...........COMPUTING THE INVENTORY COSTS FOR THE FIRST FIVE PERIODS    MIN01210
C             OF ALL FOUR MODELS                                        MIN01220
      DO 7009 KK=1,20                                                   MIN01230
      IF((XNV(KK+4)+D(KK)/2.0).LT.0.00001) GO TO 69                     MIN01240
      RIALV(KK)=(XNV(KK+4)+D(KK)/2.0)*(H(KK)*.25*C(KK))+SU(KK)+         MIN01250
     1BNIS(KK)+C(KK)*X(KK)                                              MIN01260
      GO TO 7009                                                        MIN01270
   69 RIALV(KK)=SU(KK)+BNIS(KK)+C(KK)*X(KK)                             MIN01280
 7009 TRIALV=TRIALV+RIALV(KK)                                           MIN01290
C...........COMPUTING THE INVENTORY COSTS FOR THE LAST (SIXTH) PERIOD   MIN01300
C             OF ALL FOUR MODELS                                        MIN01310
      DO 9007 KK=21,N                                                   MIN01320
      IF((XNV(KK)+X(KK)/2.0).LT.0.000001) GO TO 669                     MIN01330
      RIALV(KK)=((XNV(KK)+X(KK))/2.0)*(H(KK)*.25*C(KK))+SU(KK)+BNIS(KK)+MIN01340
     1C(KK)*X(KK)                                                       MIN01350
      GO TO 9007                                                        MIN01360
  669 RIALV(KK)=SU(KK)+BNIS(KK)+C(KK)*X(KK)                             MIN01370
 9007 TRIALV=TRIALV+RIALV(KK)                                           MIN01380
      RETURN                                                            MIN01390
      END                                                               MIN01400
```

```
C              SUBROUTINE SEARCH DECISION RULE   MINIMIZATION

      SUBROUTINE SDRMIN
      COMMON X(24),C(24),H(24),S(24),SU(24),XNV(24),D(24),BNIS(24),RIALV
     1(24),XMAX(24),XMIN(24),N,MAXTRY,TRIALV
      DIMENSION STEP(100),XBEST(100)
C INITIALIZE CONTROL PARAMETERS
      MAXABT=10
      PCTHI=.5
      PCTMED=.25
      PCTMIN=1.E-2
      STPDEC=.1
      STPINC=2.
      STPMIN=1.E-10
      STPSET=.1
C INITIALIZE COUNTERS
      IPAGE=1
      LINE=100
      NDEC=0
      NEVAL=0
      NFWDF=0
      NFWDO=0
      NFWDS=0
      NMIN=0
      NPAT=0
      NPRIOR=0
      NPROG=0
      NREST=0
      NREVF=0
      NREVO=0
      NREVS=0
C COMPUTE PATTERN MOVE PRINT FREQUENCY
      IPRINT=10/N
      IF(IPRINT.LT.1)IPRINT=1
C TERMINATE THE SEARCH IF MAXABT(SEARCH RESTART LIMIT)IS EXCEEDED
200   NABORT=NMIN+NPROG
      IF(NABORT.GT.MAXABT) GO TO 800
C RESET EXP SEARCH SUCCESS STATUS FLAG + PATTERN MOVE ACCELERATOR
      IFLAG=0
      ACCEL=2.2
C COMPUTE INITIAL STEP SIZE VECTOR STEP(I) + SAVE TRIAL VECTOR X(I)
      DO 250 I=1,N
      STEP(I)=STPSET*(XMAX(I)-XMIN(I))
250   XBEST(I)=X(I)
C TEST TO INSURE ALL SOLUTION VECTOR COMPONENTS ARE WITHIN BOUNDS
C IF OUTSIDE RESET TO XMAX(I) OR XMIN(I) BOUNDARY
300   DO 350 I=1,N
```

```
      IF(X(I).GT.XMAX(I)) X(I)=XMAX(I)
      IF(X(I).LT.XMIN(I)) X(I)=XMIN(I)
350   CONTINUE
C EVALUATE OBJECTIVE FUNCTION + INCREMENT EVALUATION COUNTER
      CALL OBJFCT
      NEVAL=NEVAL+1
C RECORD INITIAL SOLUTION VALUE FOR COMPARISON DURING EXP SEARCH PHASE
      GOODV=TRIALV
C
C IF INITIAL PASS OF SEARCH INITIALIZE + PRINT INITIAL CONDITIONS
      IF(NEVAL.GT.1) GO TO 400
      PCTDEC=0.
C INITIALIZE WITH INITIAL SOLUTION VALUE (TRIALV)
      BESTV=TRIALV
      FIRSTV=TRIALV
      PRIORV =TRIALV
C PRINT INITIAL CONDITIONS
      WRITE(8,930) IPAGE
      WRITE(8,932)
      WRITE(8,934) N
      WRITE(8,936) MAXTRY
      WRITE(8,938) MAXABT
      WRITE(8,940) STPSET
      WRITE(8,942) STPINC
      WRITE(8,944) STPDEC
      WRITE(8,946) STPMIN
      WRITE(8,948) PCTMIN
      WRITE(8,950) PCTMED
      WRITE(8,952) PCTHI
      WRITE(8,954) ACCEL
      WRITE(8,960) (X(I),I=1,N)
      WRITE(8,962) (XMAX(I),I=1,N)
      WRITE(8,964) (XMIN(I),I=1,N)
      WRITE(8,966) (STEP(I),I=1,N)
C PRINT NEW PAGE HEADING, INITIAL SEARCH VALUES + RETURN TO STMT 400
      JPRINT=1
      GO TO 900
C
C TERMINATE THE SEARCH IF MAXTRY(OBJFCT EVALUATION LIMIT) IS EXCEEDED
C
400   IF(NEVAL.GT.MAXTRY) GO TO 800
C
C INITIALIZE MINIMUM STEP SIZE COUNTER
      NSTEP=0
C
C
C          START EXPLORATORY SEARCH SECTION
C...................................................................
```

```
C
      DO 500 I=1,N
C IF STEP SIZE IS ZERO OMIT EXP SEARCH FOR THAT COMPONENT
      IF(STEP(I).EQ.0) GO TO 490
C
C FORWARD MOVE LOGIC
C-------------------------------------------
C COMPUTE X(I) FOR FORWARD MOVE
      X(I)=X(I)+STEP(I)
C IF OUTSIDE BOUNDS TRY A REVERSE MOVE
      IF(X(I).GT.XMAX(I) .OR. X(I).LT.XMIN(I)) GO TO 410
C EVALUATE OBJEVTIVE FUNCTION + INCREMENT EVALUATION COUNTER
      CALL OBJFCT
      NEVAL=NEVAL+1
C IF NEW POINT IS NOT BETTER TRY A REVERSE MOVE
      IF(TRIALV.GE.GOODV) GO TO 420
      GOODV=TRIALV
      NFWDS=NFWDS+1
C IF LAST EXP SEARCH WAS ALSO SUCCESSFUL THEN INCREASE STEP SIZE
      IF(IFLAG.GT.0) STEP(I)=STPINC*STEP(I)
      GO TO 500
C
C REVERSE MOVE LOGIC
C ------------------------------------
C
C INCREMENT COUNTERS
410   NFWDO=NFWDO+1
      GO TO 430
420   NFWDF=NFWDF+1
C COMPUTE X(I) FOR REVERSE MOVE
430   X(I)=X(I)-2.*STEP(I)
C IF OUTSIDE BOUNDS RESTORE X(I) + REDUCE STEP SIZE
      IF(X(I).GT.XMAX(I) .OR.X(I).LT.XMIN(I)) GO TO 440
C EVALUATE OBJECTIVE FUNCTION + INCREMENT EVALUATION COUNTER
      CALL OBJFCT
      NEVAL=NEVAL+1
C IF NEW POINT IS NOT BETTER RESTORE X(I) + REDUCE STEP SIZE
      IF(TRIALV.GT.GOODV) GO TO 450
      GOODV=TRIALV
C     REVERSE STEP DIRECTION
      STEP(I)=-STEP(I)
      NREVS=NREVS+1
      GO TO 500
C
C RESTORE X(I) + REDUCE STEP LOGIC
C ------------------------------------
C INCREMENT COUNTERS
440   NREVO=NREVO+1
```

```
      GO TO 460                                                         MIN02830
450   NREVF=NREVF+1                                                     MIN02840
C RESTORE X(I) TO VALUE PRIOR TO FORWARD MOVE                           MIN02850
460   X(I)=X(I)+STEP(I)                                                 MIN02860
C REDUCE STEP SIZE                                                      MIN02870
      STEP(I)=STPDEC*STEP(I)                                            MIN02880
C TEST STEP SIZE - RESTORE TO MIN VALUE(STPMIN) IF BELOW MINIMUM        MIN02890
      IF(ABS(STEP(I)).GE.STPMIN) GO TO 500                              MIN02900
      IF(STEP(I).GE.O.) STEP(I)=STPMIN                                  MIN02910
      IF(STEP(I).LT.O.) STEP(I)=-STPMIN                                 MIN02920
C INCREMENT MINIMUM STEP SIZE COUNTERS                                  MIN02930
490   NREST=NREST+1                                                     MIN02940
      NSTEP=NSTEP+1                                                     MIN02950
500   CONTINUE                                                          MIN02960
C                                                                       MIN02970
C NOTE:GOODV IS THE BEST VALUE FOUND DURING THE EXP SEARCH PHASE        MIN02980
C             THE CORRESPONDING SOLUTION VECTOR IS X(I)                 MIN02990
C                                                                       MIN03000
C                  END OF EXPLORATORY SEARCH SECTION                    MIN03010
C.....................................................................  MIN03020
C                                                                       MIN03030
C TEST FOR SUCCESS OR FAILURE OF EXPLORATORY SEARCH                     MIN03040
      IF(GOODV.LT.BESTV) GO TO 700                                      MIN03050
C                                                                       MIN03060
C                                                                       MIN03070
C                                                                       MIN03080
C  EXPLORATORY SEARCH IS A FAILURE                                      MIN03090
C-------------------------------------------------                      MIN03100
C RESTORE TRIAL VECTOR X(I) TO BEST PRIOR VALUE                         MIN03110
      DO 650 I=1,N                                                      MIN03120
650   X(I)=XBEST(I)                                                     MIN03130
C IF ALL COMPONENTS OF STEP(I) ARE AT MIN INCREMENT COUNTER AND         MIN03140
C RESTART SEARCH USING INITIAL STEP SIZE VECTOR                         MIN03150
      IF(NSTEP.LT.N) GO TO 660                                          MIN03160
C TEST FOR NEW PAGE, PRINT SEARCH DATA + GO TO STMT 200 TO RESTART      MIN03170
      NMIN=NMIN+1                                                       MIN03180
      JPRINT=2                                                          MIN03190
      GO TO 900                                                         MIN03200
C RESET VARIABLES, INCREMENT COUNTER + MAKE ANOTHER EXP SEARCH          MIN03210
660   GOODV=BESTV                                                       MIN03220
      IFLAG=0                                                           MIN03230
      NDEC=NDEC+1                                                       MIN03240
      GO TO 400                                                         MIN03250
C  EXPLORATORY SEARCH IS A SUCCESS                                      MIN03260
C   ---------------------------------                                   MIN03270
C   FIND PCTDEC-THE PERCENT DECREASE/EVALUATION SINCE LAST SUCCESSFUL   MIN03280
C   EXPLORATORY SEARCH + RESET VARIABLES                                MIN03290
700   BESTV=GOODV                                                       MIN03300
```

```
      XX=NEVAL-NPRIOR                                                   MIN03310
      PCTDEC=ABS(100.0*(PRIORV-GOODV)/(PRIORV*XX))                      MIN03320
      PRIORV=GOODV                                                      MIN03330
      NPRIOR=NEVAL                                                      MIN03340
C   TEST FOR SATISFACTORY PROGRESS DEFINED BY PCTMIN                    MIN03350
      IF(PCTDEC.GE.PCTMIN) GO TO 720                                    MIN03360
C                                                                       MIN03370
C   PROGRESS IS SATISFACTORY - SAVE SOLUTION VECTOR, PRINT STATUS +     MIN03380
C   RESTART SEARCH USING INITIAL STEP SIZE VECTOR                       MIN03390
      NPROG=NPROG+1                                                     MIN03400
      DO 710 I=1,N                                                      MIN03410
710   XBEST(I)=X(I)                                                     MIN03420
C   TEST FOR NEW PAGE, PRINT SEARCH DATA + GO TO STMT 200 TO RESTART    MIN03430
      JPRINT=2                                                          MIN03440
      GO TO 900                                                         MIN03450
C   PROGRESS IS SATISFACTORY  -  INVOKE PATTERN MOVE LOGIS              MIN03460
C   FIND PATTERN MOVE ACCELERATION FACTOR (ACCEL) FROM ADAPTIVE LOGIC   MIN03470
720   IF(PCTDEC.LT.PCTMED) ACCEL=ACCEL+0.10                             MIN03480
      IF(PCTDEC.LT.PCTHI)  ACCEL=ACCEL+0.05                             MIN03490
      IF(ACCEL.GT.3.5) ACCEL=2.2                                        MIN03500
C   SA E CURRENT SOLUTION VECTOR IN XBEST(I) + PLACE NEW PATTERN        MIN03510
C   MOVE VECTOR IN X(I) FOR EVALUATION                                  MIN03520
      DO 750 I=1,N                                                      MIN03530
      XX=XBEST(I)                                                       MIN03540
      XBEST(I)=X(I)                                                     MIN03550
750   X(I)=XX+ACCEL*(X(I)-XX)                                           MIN03560
C   SET EXP SEARCH SUCCESS FLAG + INCREMENT COUNTER                     MIN03570
      IFLAG=1                                                           MIN03580
      NPAT=NPAT+1                                                       MIN03590
      JPRINT=3                                                          MIN03600
C   TEST PRINT  CONTROL PARAMETER PRINT PARAMETER(IPRINT) + PRINT LINE OMIN03610
C  INFORMATION IF NPAT IS AN EVEN MULTIPLE OF IPRINT                    MIN03620
C   RETURN TO STMT 300 AFTER PRINTING STATUS INFO                       MIN03630
      IF(MOD(NPAT,IPRINT)) 300,900,300                                  MIN03640
C   TEST FOR NEW PAGE, PRINT LINE OF DATA + RETURN TO PRIOR STMT        MIN03650
C   ------------------------------------------------------------        MIN03660
900   IF(LINE.LE.40) GO TO 901                                          MIN03670
      IF(JPRINT.NE.1) WRITE(8,990)                                      MIN03680
C   WRITE NEW PAGE HEADING                                              MIN03690
      IPAGE=IPAGE+1                                                     MIN03700
      LINE=0                                                            MIN03710
      WRITE(8,930) IPAGE                                                MIN03720
      WRITE(8,990)                                                      MIN03730
      WRITE(8,991)                                                      MIN03740
      WRITE(8,990)                                                      MIN03750
      WRITE(8,992)                                                      MIN03760
      WRITE(8,993)                                                      MIN03770
      WRITE(8,990)                                                      MIN03780
```

```
C     WRITE LINE OF STATUS INFO                                         MIN03790
901   WRITE(8,995) NEVAL,BESTV,NPAT,NDEC,NMIN,NPROG,PCTDEC,NFWDO,NFWDS, MIN03800
     1 NFWDF,NREVO,NREVS,NREVF,NREST                                    MIN03810
      LINE=LINE+1                                                       MIN03820
      GO TO (400,200,300),JPRINT                                        MIN03830
C                                                                       MIN03840
C                                                                       MIN03850
C  SEARCH TERMINATION - RESET X(I) + PRINT TERMINATION DATA             MIN03860
C-----------------------------------------------------------------      MIN03870
800   DO 810 I=1,N                                                      MIN03880
810   X(I)=XBEST(I)                                                     MIN03890
      WRITE(8,995) NEVAL,BESTV,NPAT,NDEC,NMIN,NPROG,PCTDEC,NFWDO,NFWDS, MIN03900
     1 NFWDF,NREVO,NREVS,NREVF,NREST                                    MIN03910
      WRITE(8,990)                                                      MIN03920
      IPAGE=IPAGE+1                                                     MIN03930
      WRITE(8,930) IPAGE                                                MIN03940
      WRITE(8,970)                                                      MIN03950
      WRITE(8,972)  NEVAL                                               MIN03960
      WRITE(8,974) FIRSTV                                               MIN03970
      WRITE(8,976) BESTV                                                MIN03980
      WRITE(6,976) BESTV                                                MIN03990
      XX=NEVAL                                                          MIN04000
      PCTDEC=ABS(100.0*(FIRSTV-BESTV)/FIRSTV)                           MIN04010
      WRITE(8,977) PCTDEC                                               MIN04020
      PCTDEC=PCTDEC/XX                                                  MIN04030
      WRITE(8,978) PCTDEC                                               MIN04040
      WRITE(8,980) (XBEST(I),I=1,N)                                     MIN04050
      WRITE(6,980) (XBEST(I),I=1,N)                                     MIN04060
      WRITE(8,982) (STEP(I),I=1,N)                                      MIN04070
C                                                                       MIN04080
930   FORMAT(1H1,34X,'* * *  A D A P T I V E   P A T T E R N   S E A R  MIN04090
     1C H   * * *',27X,'PAGE',I3, //          )                         MIN04100
932   FORMAT(' I N I T I A L   C O N D I T I O N S' )                   MIN04110
934   FORMAT(5X,'NO OF INDEPENDENT VARIABLES', 47X,I10)                 MIN04120
936   FORMAT(5X,'MAX NO OF OBJECTIVE FUNCTION EVALUATIONS', 34X,I10)    MIN04130
938   FORMAT(5X,'MAX NO OF RESTARTS AFTER STEP SIZE OR UNSAT PROGRESS ABMIN04140
     1ORT', 16X,I10 )                                                   MIN04150
940   FORMAT(5X,'STEP SIZE MULTIPLIER - INITIAL START', 38X,E10.3 )     MIN04160
942   FORMAT(5X,'STEP SIZE MULTIPLIER - FWD MOVE SUCCESS FOLLOWING EXP SMIN04170
     1EARCH SUCCESS',6X,E10.3 )                                         MIN04180
944   FORMAT(5X,'STEP SIZE MULTIPLIER - FORWARD + REVERSE MOVE FAILURE',MIN04190
     1 21X,F10.3 )                                                      MIN04200
946   FORMAT(5X,'MIN STEP SIZE FOR ALL INDEPENDENT VARIABLES',31X,E10.3)MIN04210
948   FORMAT(5X,'UNSAT PROGRESS ABORT (MIN PERCENT IMPROVEMENT PER EVALUMIN04220
     1ATION)', 13X,E10.3 )                                              MIN04230
950   FORMAT(5X,'PATTERN MOVE ACCELERATOR THRESHOLD - LOW/MED PCT PER EVMIN04240
     1ALUATION', 11X,E10.3 )                                            MIN04250
952   FORMAT(5X,'PATTERN MOVE ACCELERATOR THRESHOLD - MED/HI PCT PER EVAMIN04260
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     1LUATION', 11X,E10.3 )                                             MIN04270
 954  FORMAT(5X,'INITIAL PATTERN MOVE ACCELERATOR',42X,E10.3 )          MIN04280
 960  FORMAT(  ///  ,' INITIAL SOLUTION VECTOR - X(I)',/,(10F12.4))     MIN04290
 962  FORMAT(  ///  ,' UPPER RANGE CONSTRAINT VECTOR - XMAX(I)', /,     MIN04300
     1 (10F12.4) )                                                      MIN04310
 964  FORMAT(  ///  ,' LOWER RANGE CONSTRAINT VECTOR - XMIN(I)', /,     MIN04320
     1 (10F12.4) )                                                      MIN04330
 966  FORMAT(  ///  ,' INITIAL STEP SIZE  ECTOR - STEP(I)', /,(10F12.4))MIN04340
 970  FORMAT(' S E A R C H   T E R M I N A T I O N   R E S U L T S' )   MIN04350
 972  FORMAT(5X,'NO OF OBJECTIVE FUNCTION EVALUATIONS', 8X,I12)         MIN04360
 974  FORMAT(5X,'INITIAL VALUE OF OBJECTIVE FUNCTION', 9X,E12.5 )       MIN04370
 976  FORMAT(5X,'FINAL   VALUE OF OBJECTIVE FUNCTION', 9X,E12.5 )       MIN04380
 977  FORMAT(5X,'OVERALL PERCENT DECREASE',20X,E12.5)                   MIN04390
 978  FORMAT(5X,'OVERALL PERCENT DECREASE PER EVALUATION', 5X,E12.5)    MIN04400
 980  FORMAT(  ///   ,' FINAL SOLUTION VECTOR - STEP(I) AND X(I)',      MIN04410
     1  /, (10F12.4) )                                                  MIN04420
 982  FORMAT(  ///   ,' FINAL STEP SIZE VECTOR - STEP(I)', /,(10F12.4)) MIN04430
 990  FORMAT(1X, 129(1H-) )                                             MIN04440
 991  FORMAT(' |S D R  S U M M A R Y | E X P L O R A T O R Y  S E A R CMIN04450
     1 H   D A T A | F O R W A R D  M O V E | R E V E R S E  M O V E |REMIN04460
     2STOPE|'  )                                                        MIN04470
 992  FORMAT(' | OBJ FCT   OBJ FCT   |PATTERN DEC STEP MIN STEP NO PROG MIN04480
     1   PCT DEC    |OUTSIDE   TRIAL   TRIAL |OUTSIDE   TRIAL   TRIAL |STMIN04490
     2EP AT|' )                                                         MIN04500
 993  FORMAT(' |  EVAL       VALUE     | MOVE    SEARCH   ABORT    ABORT MIN04510
     1    /EVAL    | RANGE   SUCCESS FAILURE| RANGE   SUCCESS FAILURE|MIMIN04520
     2NIMUM|' )                                                         MIN04530
 995  FORMAT(' |',I5,E15.5,2H |,I5,3I9,E14.5,2H |,I6,2I8,3H  |,I6,2I8,  MIN04540
     1 3H  |,I5,'  |')                                                  MIN04550
      RETURN                                                            MIN04560
      END                                                               MIN04570
```

LIST OF REGERENCES

1. Hadley, G. and Whitin, T. M., Analysis of Inventory Systems, Prentice-Hall, 1963.

2. Buffa, E. S. and Taubert, W.H., Production-Inventory Systems: Planning and Control, Richard D. Irwin, Inc., 1972.

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13. ABSTRACT

A study was conducted of the inventory operations of a small production plant. Based on parameters established or estimated during the study, a simple inventory model which yields stock levels for the raw material inventory and production quantities for the manufacturing operation was developed. Price comparisons were made showing the recommended policies to be more economical than those policies used before the study was conducted. A search routine proffered by Buffa and Taubert was examined and shown not to be acceptable as a method of determining production quantities for the company investigated.

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